Multichannel Kondo Effect in an Interacting Electron System: Exact Results for the Low-Temperature Thermodynamics

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We study the low-temperature thermodynamics of a spin-S magnetic impurity coupled to $m \geq 2$ degenerate bands of interacting electrons in one dimension. By exploiting boundary conformal field theory techniques, we derive exact results for the *possible* impurity thermal and magnetic response. The leading behavior of the impurity magnetic susceptibility is shown to be insensitive to the electron-electron interaction. In contrast, there are two types of scaling behavior of the impurity specific heat consistent with the symmetries of the problem: *Either* it remains the same as for the ordinary multichannel Kondo problem for noninteracting electrons *or* it acquires a new leading term governed by a critical exponent $\alpha_m = (K_{\rho,m}^{-1} - 1)/m$, where $K_{\rho,m} \leq 1$ is a generalized (m−channel) Luttinger liquid charge parameter measuring the strength of the repulsive electron-electron interaction. We conjecture that the latter behavior is indeed realized when the impurity is exactly screened ($m = 2S$).

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I. INTRODUCTION

The violation of Fermi-liquid behavior in the normal state of the high-T_c superconductors has turned the study of *non-Fermi liquid* phenomena into a major theme in condensed matter physics [1]. Additional motivation comes from a growing number of experimental realizations of low-dimensional electron structures - such as quasi one-dimensional (1D) organic conductors [2], or point contact tunneling in fractional quantum Hall devices [3] - where electron correlations are seen to produce manifest non-Fermi liquid behavior.

Two prominent model problems, serving as paradigms for the study of non-Fermi liquids, are the *Luttinger liquid* [4,5] and the (overscreened) *multichannel Kondo effect* [6–9]. "Luttinger liquid" is the code name for the universal low-energy behavior of interacting electrons in 1D, whereas the multichannel Kondo model describes a magnetic impurity coupled via a spin exchange to several degenerate bands of non-interacting electrons in 3D. Both problems have yielded to exact solutions, exhibiting a wealth of properties not contained in the standard Fermi liquid picture of the metallic state.

Here we consider a spin-S magnetic impurity coupled to $m \geq 2$ degenerate bands of *interacting* electrons in 1D, thus extending the ordinary multichannel Kondo model to the case of interacting electrons. Specifically, we address the question about the influence of electron-electron interaction on the low-temperature thermal and magnetic response of the impurity-electron (screening-cloud) composite. This problem is particularly interesting as it involves the interplay between two kinds of electron correlations; one induced by the spin exchange interaction with the impurity, the other coming from the direct electron-electron interaction. Moreover, the study of a magnetic impurity in the presence of interacting electrons may shed new light on possible experimental realizations of the multichannel Kondo effect, such as certain Uranium-containing heavy-fermion materials [10], or the coupling of conduction electrons to structural defects in metal point contacts [11].

We study the problem using boundary conformal field theory (BCFT) [12], assuming that at low temperatures the impurity-electron interaction renormalizes onto a scale-invariant boundary condition on the bulk theory. This approach, first suggested by Affleck and Ludwig for the ordinary multichannel Kondo problem [8,13], has successfully been employed for a single channel of interacting electrons coupled to a spin-1/2 impurity [14,15]. In the present case, with an arbitrary number of electron channels $m \geq 2$, a BCFT analysis allows for a complete classification of all *possible* critical behaviors of the impurity-electron composite. Being exact, this information should prove useful as a guide to - and a test of the validity of - other, more direct approaches to the problem (yet to be carried out). Specifically, we shall show that conformal invariance together with the internal symmetries of the problem restrict the possible types of critical behavior to only two: *Either* the theory is the same as for noninteracting electrons *or* the electron interaction generates a specific boundary operator that produces a new leading behavior of the impurity thermal response. In both cases the leading impurity magnetic response is insensitive to the electron-electron interaction, implying that the screening of the impurity is realized in the same way as in the noninteracting problem, with over-, exact, or underscreening depending on the number of channels and the magnitude of the impurity spin. While our method cannot pinpoint which of the two scenarios is actually realized, we conjecture - guided by analogous results for the one-channel problem [16,17] - that electron interactions in the bulk *do* induce an anomalous term in the impurity specific heat in the case of exact screening ($m = 2S$).

II. THE MODEL

As microscopic bulk model we take a multiband Hubbard chain with repulsive on-site interaction $U > 0$,

$$H_{el} = -t \sum_{n,i,\sigma} (c_{n,i\sigma}^{\dagger} c_{n+1,i\sigma} + h.c.) + U \sum_{n,ij,\mu\sigma} n_{n,i\sigma} n_{n,j\mu}, \tag{1}$$

where $c_{n,i\sigma}$ is the electron operator at site n, with $i = 1,....,m$ and $\sigma = \uparrow, \downarrow$ band- and spin indices, respectively, and $n_{n,i\sigma} = c_{n,i\sigma}^{\dagger} c_{n,i\sigma}$ is the number operator. This model - and its variants - has been extensively studied in the literature, most recently in [18] where it was argued that enhanced superconducting fluctuations may result when the degeneracy of the on-site interband coupling U is properly lifted. At large wavelengths we can perform a continuum limit $c_{n,i\sigma} \to \sqrt{\frac{a}{2\pi}} \Psi_{i\sigma}(na)$ (with a the lattice spacing), which, for small U and away from half-filling, takes (1) onto

$$H_{el} = \frac{1}{2\pi} \int dx \left\{ v_F \left[:\psi_{L,i\sigma}^{\dagger}(x) i \frac{d}{dx} \psi_{L,i\sigma}(x): - :\psi_{R,i\sigma}^{\dagger}(x) i \frac{d}{dx} \psi_{R,i\sigma}(x): \right] \right.$$

$$+\frac{g}{2} :\psi^\dagger_{r,i\sigma}(x)\psi_{r,i\sigma}(x)\psi^\dagger_{s,j\mu}(x)\psi_{s,j\mu}(x): +g :\psi^\dagger_{L,i\sigma}(x)\psi_{R,i\sigma}(x)\psi^\dagger_{R,j\mu}(x)\psi_{L,j\mu}(x): \Big\}. \tag{2}$$

Here $\psi_{L/R,i\sigma}(x)$ are the left/right moving components of the electron field $\Psi_{i\sigma}(x)$, expanded about the Fermi points $\pm k_F$: $\Psi_{i\sigma}(x) = e^{-ik_F x}\psi_{L,i\sigma}(x) + e^{ik_F x}\psi_{R,i\sigma}(x)$. Summation over repeated indices for band, spin, and chirality $r, s = R, L$ is implied. The normal ordering is defined w.r.t. the filled Dirac sea, and v_F and g are given by $v_F = 2at\sin(ak_F)$ and $g = Ua/\pi$ respectively.

For the purpose of implementing BCFT techniques in the presence of a Kondo impurity (yet to be added), we decouple charge-, spin-, and band- *(flavor-)* degrees of freedom in (2) by a Sugawara construction [19], using the $U(1)$ (charge), $SU(2)_m$ (level m, spin), and $SU(m)_2$ (level 2, flavor) Kac-Moody currents:

$$J_r(x) = :\psi^\dagger_{r,i\sigma}(x)\psi_{r,i\sigma}(x): \tag{3}$$

$$\boldsymbol{J}_r(x) = :\psi^\dagger_{r,i\sigma}(x)\frac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}\psi_{r,i\mu}(x): \tag{4}$$

$$J^A_r(x) = :\psi^\dagger_{r,i\sigma}T^A_{ij}\psi_{r,j\sigma}:, \tag{5}$$

where $\boldsymbol{\sigma}$ are the Pauli matrices, and $T^A, A \in \{1, \ldots, m^2 - 1\}$, are generators of the defining representation of $SU(m)$ with normalization $\mathrm{tr}T^A T^B = 1/2\delta^{AB}$. Diagonalizing the charge sector by a Bogoliubov transformation $J_{L/R} = \cosh(\theta)j_{L/R} - \sinh(\theta)j_{R/L}$, with $\coth(2\theta) = 1 + v_F/((2m-1)g)$, we obtain the critical bulk Hamiltonian

$$H^*_{el} = \frac{1}{2\pi}\int dx \left\{ \frac{v_c}{4m} :j^i_L(x)j^i_L(x): + \frac{v_s}{m+2} :\boldsymbol{J}^i_L(x)\cdot\boldsymbol{J}^i_L(x): + \frac{v_f}{m+2} :J^{iA}_L(x)J^{iA}_L(x): \right\}, \tag{6}$$

where $v_c = v_F(1 + 2(2m-1)g/v_F)^{1/2}, v_s = v_f = v_F - g$. We have here retained only exactly marginal terms in the interaction by removing two (marginally) irrelevant terms in the spin and flavor sectors [20]. We have also replaced the right-moving currents with a second species (labeled by "2") of left-moving currents: $j^2_{L/R}(x) \equiv j_{R/L}(-x)$ for $x > 0$ (with $j^1_{L/R}(x) \equiv j_{L/R}(x)$), and analogously for spin and flavor currents. This amounts to folding the system onto the positive x-axis, with a boundary condition

$$j^{1/2}_L(0) = j^{2/1}_R(0) \tag{7}$$

at the origin, and then analytically continuing the currents back to the full $x-$axis. Here (7) simulates the continuity at $x = 0$ of the original bulk theory (with the analogous boundary conditions in spin- and flavor sectors). The Sugawara form of H^*_{el} in (6) implies invariance under independent $U(1)^i$, $SU(2)^i_m$, and $SU(m)^i_2$ transformations (with $i = 1, 2$ labeling the two species), reflecting the *chiral* symmetry of the critical bulk theory.

We now insert a local spin $\boldsymbol{S}$ at $x = 0$, and couple it to the electrons by an antiferromagnetic ($\lambda > 0$) spin exchange interaction

$$H_K = \lambda : (\psi^\dagger_{L,i\sigma}(0) + \psi^\dagger_{R,i\sigma}(0))\frac{\boldsymbol{\sigma}_{\sigma\mu}}{2}(\psi_{L,i\mu}(0) + \psi_{R,i\mu}(0))\cdot\boldsymbol{S}: . \tag{8}$$

In the low-energy limit the impurity is expected [13] to renormalize to a conformally invariant boundary condition on the bulk theory, changing (7) into a new nontrivial boundary condition on H^*_{el}. By using BCFT to extract the set of *boundary operators* present for this boundary condition, finite-temperature effects due to the impurity can be accessed via standard finite-size scaling by treating (Euclidean) time as an inverse temperature.

III. IMPURITY CRITICAL BEHAVIOR

The set of boundary operators $\mathcal{O}_j$ and the corresponding *boundary scaling dimensions* Δ_j can be derived from the finite-size spectrum of H^*_{el} through a conformal mapping from the half-plane to a semi-infinite strip. The mapping is such that the boundary condition corresponding to the impurity on the half-plane is mapped to both sides of the strip, with the boundary operators in the plane in one-to-one correspondence to the eigenstates on the strip. In particular, the boundary dimensions are related to the energy spectrum through the relation $E = E_0 + \pi v\Delta/\ell$, with E_0 the groundstate energy, and ℓ the width of the strip [12].

A complete set of eigenstates of H^*_{el} are given by the charge-, spin- and flavor *conformal towers* , each tower defined by a *Kac-Moody primary state* and its descendants [19]. The primaries are labeled by $U(1)$ quantum numbers q^i

in the charge sector, $SU(2)$ quantum numbers j^i in the spin sector and $SU(m)$ quantum numbers *(Dynkin labels)* $(m_1^i, \ldots, m_{m-1}^i)$ in the flavor sector:

$$q^{\frac{1}{2}} = C^{\frac{1}{2}} \frac{e^{\theta}}{2} \pm D^{\frac{1}{2}} \frac{e^{-\theta}}{2}, \quad j^i = 0, \frac{1}{2}, ..., \frac{m}{2}, \quad \sum_{k=1}^{m-1} m_k^i = 0, 1, 2 , \tag{9}$$

where $C^i, D^i \in \mathbb{Z}$ and $m_k^i \in \mathbb{N}$, with $i = 1, 2$ labelling the two species as above. We can express the complete energy spectrum, and consequently the complete set of possible boundary scaling dimensions $\Delta = \Delta^1 + \Delta^2$ in terms of the quantum numbers in (9):

$$\Delta^i = \frac{(q^i)^2}{4m} + \frac{j^i(j^i + 1)}{m + 2} + \frac{1}{2(m+2)} \sum_{k=1}^{m-1} m_k^i \left(f_m(k,k) + \sum_{l=1}^{m-1} m_l^i \frac{f_m(k,l)}{m} \right) + \mathcal{N}, \tag{10}$$

where $f_m(k,l) = \min(k,l)(m - \max(k,l))$ and $\mathcal{N} \in \mathbb{N}$.

Each conformal boundary condition corresponds to a *selection rule* which specifies that only certain combinations of conformal towers are allowed. Since the trivial boundary condition (7) simply defines the bulk theory in terms of a boundary theory, the associated selection rule reproduces the *bulk* scaling dimensions of H_{el}^* [14]. It is less obvious how to identify the correct selection rule for the nontrivial boundary condition representing (8). Fortunately, we do not need the full selection rule to extract the leading impurity critical behavior. For this purpose it is sufficient to identify the *leading irrelevant boundary operator* [8] (LIBO) that can appear in the scaling Hamiltonian, as this is the operator that drives the dominant response of the impurity. As the possible correction-to-scaling-operators are boundary operators constrained by the symmetries of the Hamiltonian, this sets our strategy: We consider *all* selection rules for combining conformal towers in (9) (thus exhausting all conceivable boundary fixed points), for each selecting the corresponding LIBO, using (10). We then extract the *possible* impurity critical behaviors by identifying those LIBOs that (i) *produce a noninteracting limit $g \to 0$ consistent with known results*, and (ii) *respect the symmetries of* $H_{el}^* + H_K$.

A. Overscreening: $m > 2S$

Let us first focus on the case $m > 2S$. Here the noninteracting ($g = 0$) problem renormalizes to a nontrivial fixed point, as can be seen by passing to a basis of definite-parity ($P = \pm$) fields $\psi_{\pm,i\sigma}(x) = (1/\sqrt{2})(\psi_{L,i\sigma}(x) \pm \psi_{R,i\sigma}(-x))$ [21]. In this basis $H_{el}^*[g = 0] + H_K$ becomes identical to the Hamiltonian representing 3D noninteracting electrons in $2m$ channels ($P = \pm, i = 1, ...m$), coupled to a local spin in the m positive parity channels only. At low temperatures this system flows to the overscreened m−channel Kondo fixed point with a LIBO of dimension $\Delta = (4 + m)/(2 + m)$ [22]. Consider first the case that this fixed point is stable against perturbations in g (or connected to a line of $g > 0$ boundary fixed points via an exactly marginal operator). To search for a novel *leading* scaling behavior for $g \neq 0$ it is then sufficient to search for boundary operators with dimensions in the interval $1 \leq \Delta \leq (4+m)/(2+m)$ which produce an impurity response analytically connected to that of the non-interacting theory. An operator with $1 \leq \Delta < 3/2$ contributes an impurity specific heat scaling as $(\Delta - 1)^2 T^{2\Delta - 2}$ [8], and condition (i) then requires that, as $g \to 0$, $\Delta_{LIBO} \to (4 + m)/(2 + m)$ *or* that $\Delta_{LIBO} \to 1$ (with in this case next-leading dimension $\Delta \to (4+m)/(2+m)$). On the other hand, if the $g = 0$ fixed point gets destabilized as g is switched on, condition (i) enforces the LIBO at the new $g > 0$ boundary fixed point to become marginally relevant for $g = 0$ (so as to produce the necessary flow back to the known $g = 0$ overscreened m−channel fixed point) [23]. Thus, for this case condition (i) unambiguously requires that $\Delta_{LIBO} \to 1$ as $g \to 0$.

Turning to condition (ii), we note that the Kondo interaction (8) couples L and R fields and thus breaks the chiral gauge invariance in all three sectors. This implies that the Kac-Moody symmetries gets broken down to their diagonal subgroups, i.e. $U(1)^1 \times U(1)^2 \to U(1)$ in the charge sector, $SU(2)_m^1 \times SU(2)_m^2 \to SU(2)_{2m}$ in the spin sector, and $SU(m)_2^1 \times SU(m)_2^2 \to SU(m)_4$ in the flavor sector. Operators with non-zero values of q_1 and q_2 may thus appear in the charge sector, provided that $q_1 = -q_2$ as required by conservation of total charge. Similarly, operators in spin- and flavor sectors with non-zero quantum numbers j^i and m_j^i are now allowed, provided that they transform as singlets under the diagonal subgroups. Remarkably, *there exists precisely one generic class of g-dependent operators which satisfy condition (i) and (ii)*. It is given by

$$\mathcal{O}_1 \sim \, : \exp(\frac{i\sqrt{\pi}}{2mK_{\rho,m}} \phi_L^1): \, \times \, : \exp(\frac{i\sqrt{\pi}}{2mK_{\rho,m}} \phi_L^2): \, \times \varphi^s \times \varphi^f \tag{11}$$

of dimension $\Delta_{\mathcal{O}_1} = 1 + (K_{\rho,m}^{-1} - 1)/2m \to 1$ as $g \to 0$. Here $\phi_L^i(x)$ is a chiral charge boson of species i (i.e. $\phi_L^i(x) = \int dx\ j_L^i(x)$), while φ^s and φ^f are the singlet fields (under the diagonal subgroups) in the decomposition of the product of primary fields, $(j^1 = 1/2) \times (j^2 = 1/2)$ and $(1, 0, ..., 0) \times (0, 0, ..., 1)$ in spin- and flavor sectors, respectively. These carry dimensions $\Delta_{\varphi^s} = 3/(2(m+2))$ and $\Delta_{\varphi^f} = (m^2 - 1)/(m(m+2))$. The parameter $K_{\rho,m}$ is given by

$$K_{\rho,m} = (1 + 2(2m - 1)\frac{g}{v_F})^{-\frac{1}{2}} = \frac{v_F}{v_c} \leq 1 \tag{12}$$

and plays the role of a generalized (channel-dependent) Luttinger liquid parameter [5].

The next-leading generic irrelevant boundary operator satisfying (i) and (ii) is independent of g, and given by

$$\mathcal{O}_2 \sim \boldsymbol{J}_{-1}^1 \cdot \boldsymbol{\phi}^1 \times \mathbb{1}^2 + \mathbb{1}^1 \times \boldsymbol{J}_{-1}^2 \cdot \boldsymbol{\phi}^2, \tag{13}$$

with $\boldsymbol{J}_{-1}^i \cdot \boldsymbol{\phi}^i$ the first Kac-Moody descendant of the spin-1 primary field $\boldsymbol{\phi}^i$, obtained by contraction with the Fourier mode $\boldsymbol{J}_{-1}^i$ of the $SU(2)_m$ currents. It carries dimension $\Delta_{\mathcal{O}_2} = (m+4)/(m+2)$, and is the same operator that drives the leading impurity response in the non-interacting problem. For certain special values of m additional g-dependent boundary operators satisfying (i) and (ii) appear, but as these are non-generic and, for given m, of higher dimensions than $\Delta_{\mathcal{O}_1}$, we do not consider them here [24].

Piecing together the results, it follows that either $\mathcal{O}_1$ in (11) *or* $\mathcal{O}_2$ in (13) plays the role of a LIBO at the $g > 0$ boundary fixed point. We may thus define a scaling Hamiltonian

$$H_{scaling} = H_{el}^* + \mu_1 \mathcal{O}_1(0) + \mu_2 \mathcal{O}_2(0) + \mu_3 \mathcal{O}_3(0)..., \tag{14}$$

with μ_j conjugate scaling fields, and $\mathcal{O}_{j>2}$ less relevant operators. In the case that $\mathcal{O}_1$ in (11) does *not* appear, $\mu_1 \equiv 0$. Using (14), the thermal response may now be calculated perturbatively in the scaling fields μ_j, using standard techniques [8]. We thus obtain for the impurity specific heat:

$$C_{imp} = c_1 (1 - K_{\rho,m}^{-1})^2 T^{(K_{\rho,m}^{-1} - 1)/m} + \begin{cases} c_2 T \ln(\frac{T_K}{T}) + ... & m = 2, S = \frac{1}{2} \\ c_2' T^{\frac{4}{m+2}} + ... & m > 2, m > 2S \end{cases} \quad T \to 0. \tag{15}$$

Here c_1, c_2 and c_2' are amplitudes of second order in the scaling fields with corresponding indices, T_K plays the role of a Kondo temperature, and "..." denotes subleading terms. Note that the amplitude of the leading term in (15) always vanishes when $g = 0$, thus making the second $g-$independent term dominant.

The result in (15) is exact and independent of the precise nature of the boundary fixed point. In the case that $\mu_1 \equiv 0$, and hence $c_1 \equiv 0$, the $g > 0$ fixed point is the *same* as for the ordinary (noninteracting) overscreened problem, although the content of subleading irrelevant operators $\mathcal{O}_{j>2}$ may differ. In the alternative case, with $\mu_1 \neq 0$ ($c_1 \neq 0$), the situation is more intricate, with, in principle, three possibilities: (a) the $g = 0$ (ordinary overscreened Kondo) and $g \neq 0$ fixed points are the same, but with different contents of irrelevant operators, (b) the $g = 0$ and $g > 0$ fixed points are continuosly connected via a critical line by an exactly marginal boundary operator with a scaling field parameterized by g, or (c) the $g = 0$ and $g > 0$ fixed points are distinct and the flow between them is governed by a marginally relevant operator [23]. We postpone a discussion of the various possibilities to the next section.

Turning to the impurity magnetic susceptibility χ_{imp}, its leading scaling behavior is produced by the lowest-dimension boundary operator which contains a *nontrivial* singlet $SU(2)_{2m}$ factor that couples to the total spin density [14]. Since $\mathcal{O}_1$ in (11) only contains the identity in the $SU(2)_{2m}$ sector the desired operator is identified as $\mathcal{O}_2$ in (13). Thus, the *leading* term in the magnetic susceptibility χ_{imp} due to the impurity is independent of the electron-electron interaction, and remains the *same* as for the noninteracting overscreened $m-$channel Kondo problem [7,8]:

$$\chi_{imp} = \begin{cases} c_2 \ln(\frac{T_K}{T}) + ... & m = 2, S = 1/2 \\ c_2' T^{\frac{2-m}{m+2}} + ... & m > 2, m > 2S \end{cases} \quad T \to 0 \tag{16}$$

where c_2 and c_2' are second order in scaling fields and "..." denotes subleading terms. We find that there are no subleading interaction-dependent divergent contributions possible, as these would give rise to new divergences also in the noninteracting limit.

B. Exact screening and underscreening: $m \leq 2S$

An analysis analogous to the one above can be carried out for $m \leq 2S$ as well. Again passing to a definite-parity basis and exploring known results [9], one verifies that the *noninteracting* 1D electron groundstate carries a spin $S - m/2$ corresponding to a strong coupling fixed point. When $m = 2S$ the impurity is completely screened and the situation is essentially the same as for the ordinary single-channel Kondo problem with impurity spin $1/2$: the electron screening cloud behaves as a local Fermi liquid with a $\pi/2$ phase shift of the single-electron wave functions. Analogous to the single-channel problem [14], there are three degenerate LIBOs for this case, given by the energy-momentum tensors (of dimension $\Delta = 2$) in charge-, spin-, and flavor sectors. When $m = 2S$ these produce the *leading* term in the impurity specific heat, $C_{imp} = b_1 T + ...$, as well as in the susceptibility, $\chi_{imp} = b_2 - b_3 T^2 + ...$, with $b_{1,2,3} > 0$ amplitudes linear in the scaling fields, and with higher powers in temperature coming from higher-order descendants of the identity operator. When $m < 2S$ the impurity spin is only partially screened as there are not enough conduction electron channels to yield a singlet groundstate. This leaves an asymptotically decoupled spin $S - m/2 > 0$, adding a Curie-like contribution to C_{imp} and χ_{imp}, in addition to logarithmic corrections characteristic of asymptotic freedom [9].

Let us study the case $m = 2S$ and explore what happens when turning on the electron interaction. Implementing condition (i) from the previous section, possible LIBOs appearing for $g > 0$ must have dimensions Δ with the property $\Delta \to 1$ or $\Delta \to 2$ as $g \to 0$. Using condition (ii), we find that in addition to $\mathcal{O}_1$ in (11) there are several new allowed classes of g−dependent generic boundary operators, all with $\Delta \to 2$ as $g \to 0$. As any boundary operator with dimension $\Delta > 3/2$ produces the same *leading* scaling in temperature as a $\Delta = 2$ operator (although of different amplitudes) [14], the only *possible* leading term with an interaction-dependent exponent is again generated by $\mathcal{O}_1$, as in the overscreened case. Thus, since $\mathcal{O}_1$ does not contribute to the impurity susceptibility, its leading behavior remains the same as in the non-interacting problem, exhibiting exact screening with a constant zero-temperature contribution,

$$\chi_{imp} = b_2 + ... - b_3 T^2 + ... \; , \quad m = 2S, \; g \geq 0, \; T \to 0 \tag{17}$$

where "...." denotes possible second-order contributions in scaling fields. The amplitude b_2 is the same as in the noninteracting problem, while b_3 may pick up second-order interaction-dependent terms contributed by subleading operators. The leading *possible* interaction-dependent correction to (17) scales as

$$\chi_{imp}^{corr} \sim (K_{\rho,m}^{-1} - 1)T^{1+(K_{\rho,m}^{-1}-1)/m} \tag{18}$$

and is produced at second order by the composite boundary operator

$$\mathcal{O}_3 \sim \, :\exp(\frac{i\sqrt{\pi}}{2mK_{\rho,m}}\phi_L^1): \times \, :\exp(\frac{i\sqrt{\pi}}{2mK_{\rho,m}}\phi_L^2): \times \boldsymbol{J}_{-1}^{diag} \cdot \boldsymbol{\varphi}^s \times \varphi^f. \tag{19}$$

Here $\boldsymbol{J}^{diag} \equiv \boldsymbol{J}_L^1 + \boldsymbol{J}_L^2$ is the generator of the diagonal $SU(2)_{2m}$ subgroup in the spin sector, $\boldsymbol{\varphi}^s$ is a diagonal spin-1 field in the product of primaries $(j^1 = 1/2) \times (j^2 = 1/2)$, and the charge and flavor factors are the same as for $\mathcal{O}_1$ above. The operator $\mathcal{O}_3$ has scaling dimension $\Delta_3 = 1 + \Delta_1 = 2 + (K_{\rho,m}^{-1} - 1)/2m$ and, as seen in (18), gives a vanishing amplitude at $g = 0$, thus ensuring the correct behavior in the noninteracting limit. Whether $\mathcal{O}_3$ appears in the spectrum or not, however, must be checked by an independent method.

Two possible scenarios again emerge for the scaling of the impurity specific heat: *Either* it remains the same as in the noninteracting exactly screened problem *or*, in the case that $\mathcal{O}_1$ appears as a LIBO:

$$C_{imp} = c_1(1 - K_{\rho,m}^{-1})^2 T^{(K_{\rho,m}^{-1}-1)/m} + b_1 T + ... \; , \quad m = 2S, \; g \geq 0, \; T \to 0 \tag{20}$$

where the amplitude c_1 is independent of $K_{\rho,m}$, and b_1 may differ from the noninteracting problem by second order additive terms coming from interaction-dependent subleading corrections. Notably, by putting $m = 1$ in (20) we recover the critical exponent conjectured by Furusaki and Nagaosa [16] for the impurity specific heat in the (exactly screened) single-channel problem. The leading *possible* interaction-dependent correction to (20) is also produced by $\mathcal{O}_3$ and scales as

$$C_{imp}^{corr} \sim (K_{\rho,m}^{-1} - 1)T^{2+(K_{\rho,m}^{-1}-1)/m}, \tag{21}$$

with a vanishing amplitude in the noninteracting limit.

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For $m < 2S$, the influence of the electron-electron interactions on the impurity-electron composite corresponding to the screened part of the impurity spin is the same as for exact screening, with the same scenarios for the critical behavior. However, the weak coupling between the uncompensated (asymptotically free) part of the impurity spin and the conduction electrons produce corrections at finite T [9] that may get modified by the electron-electron interaction. We have not attempted to include these effects here.

IV. DISCUSSION

To conclude, we have presented an analysis of the possible low-temperature thermodynamics of the multichannel Kondo problem for an interacting electron system in 1D. While the leading term in the impurity susceptibility remains the same as for noninteracting electrons (for any number of channels m and impurity spin S), there are two possible behaviors for the impurity specific heat consistent with the symmetries of the problem: *Either* it remains the same as in the noninteracting problem *or* it acquires a new leading term, scaling with a non-Fermi liquid exponent $\alpha_m = (K_{\rho,m}^{-1} - 1)/m$, with $K_{\rho,m} \leq 1$ in (12) measuring the strength of the repulsive electron-electron interaction. These results are exact, given the existence of a stable boundary fixed point, an assumption common to all applications of BCFT to a quantum impurity problem [13]. As we discussed in Sec. III. A, this fixed point (for given m and S) is disconnected from that of the noninteracting problem only if the LIBO $\mathcal{O}_1$ in (11) turns marginally relevant as g is put to zero. Does this happen? A conclusive answer would require a construction of the corresponding exact renormalization group equations. This is a nontrivial task, and we have only carried out a perturbative analysis to second order in the scaling fields. Turning (14) into a Lagrangian, and integrating out the short-time degrees of freedom using the operator product expansion [25], one obtains the 1-loop RG equation for the scaling field μ_1 conjugate to $\mathcal{O}_1$:

$$\frac{d\mu_1}{d\ln\tau_0} = -(q_1 + q_2)(\lambda_1 + \lambda_2)\mu_1 \ , \tag{22}$$

with τ_0 a short-time cut-off, q_1 and q_2 the charge quantum numbers of $\mathcal{O}_1$, and λ_1 and λ_2 the scaling fields of the exactly marginal charge currents j_L^1 and j_L^2 (allowed due to breaking of particle-hole symmetry in the microscopic Hamiltonian (1) [26]). Conservation of total charge $q_1 + q_2 = 0$ thus gives $d\mu_1/d\ln\tau_0 = 0$ to second order in the scaling fields. If this property does persist to higher orders (as suggested by the cancellation of the 1-loop contribution due to a symmetry), the $g \neq 0$ and $g = 0$ boundary fixed points are either a) the same (but with different contents of irrelevant operators) *or* b) connected via a line of fixed points by the exactly marginal charge currents j_L^1 and j_L^2 (with scaling fields λ_1 and λ_2 parameterized by g). As we are unable to determine the actual scenario, the question about the relation of the $g > 0$ fixed point to that of the noninteracting problem remains open.

A second open issue is whether the electron-electron interaction influences the impurity-electron (screening cloud) composite differently when the impurity is overscreened ($m > 2S$) as compared to under- or exact screening ($m \leq 2S$). In the overscreened case with free electrons the impurity induces a critical behavior where the size of the screening cloud diverges as one approaches zero temperature. As a consequence, *all* conduction electrons become correlated due to the presence of the impurity. By turning on a weak (screened) Coulomb interaction among the electrons (in 1D simulated by the local e-e interaction in (2)) these correlations may change. Will the change be such as to produce a novel impurity critical behavior? While our analysis does not provide an answer, it predicts its exact form if it does appear. In the case of exact screening there is strong evidence that the impurity scaling behavior is indeed governed by interaction-dependent exponents. In this case the screening cloud has a finite extent. Turning on the Coulomb interaction, electrons "outside" of the cloud will become correlated, most likely influencing the rate with which they tunnel into and out of the cloud, hence influencing its properties. As this impurity-electron composite is described by a Fermi-liquid fixed point (where the electrons simply acquire a phase shift) in exact analogy with the single-channel Kondo problem, we expect that the effect of turning on electron interactions will indeed be similar to the single-channel case. Considering the recent Monte Carlo data by Egger and Komnik [17] supporting the single-channel Furusaki-Nagaosa scaling [16], this strongly favors the appearance of the interaction-dependent exponents in (18), (20) and (21) when $m = 2S$.

Acknowledgment

[1] For a review, see e.g. H. J. Schulz in *Proceedings of Les Houches Summer School LXI,* eds. E. Akkermans *et al.* (Elsevier, Amsterdam, 1995).

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[19] For a review, see e.g. J. Fuchs, *Affine Lie Algebras and Quantum Groups*, (Cambridge University Press, Cambridge 1992) and references therein.

[20] The irrelevance (in renormalization group sense) of the excluded piece $H_{excl} = g(: \psi^\dagger_{L,i\sigma}\psi_{R,i\sigma}\psi^\dagger_{R,j\mu}\psi_{L,j\mu} : + \frac{1}{2m} : \psi^\dagger_{L,i\sigma}\psi_{L,i\sigma}\psi^\dagger_{R,j\mu}\psi_{R,j\mu} :)$ in (6) can be seen by using the fact that H_{el} in (2) has a $U(1)\times SU(2m)$ symmetry, and hence can be expressed in terms of $U(1)$ and $SU(2m)_1$ Kac-Moody currents. In terms of these $H_{excl} = -2g J_L^A J_R^A$, $A \in \{1,\ldots,(2m)^2-1\}$, which is (marginally) irrelevant for $g > 0$. (For a proof of this in the analogous $SU(2)_1$ case, see I. Affleck, D. Gepner, H. J. Schulz and T. Ziman, J. Phys. A: Math. Gen. **22**, 511 (1989).)

[21] Note that the interacting part of the bulk Hamiltonian H_{el} becomes nonlocal in the definite-parity basis, thus precluding its use when $g \neq 0$.

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[23] An inspection of possible selection rules for a $g > 0$ stable boundary fixed point, using (10), shows that only dimensions $\Delta \geq 1$ are allowed in the limit $g \to 0$. As no symmetry is broken at $g = 0$, a destabilizing operator can hence be at most *marginally* relevant.

[24] A computer check of the spectra with $2 \leq m \leq 50$, using (10), reveals that additional operators satisfying (i) and (ii) occur at $m = 2, 4, 6, 8, 9, 18, 32$ and 50. As there is no change in the structure of the Hamiltonian for these particular channel numbers, we expect these operators to be excluded by the selection rule defining the boundary fixed point.

[25] For a review of the method, see e.g. J. L. Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge University Press, Cambridge 1996).

[26] Although the multiband Hubbard model in (1) is not invariant under charge conjugation (particle-hole transformation) away from half-filling, the low-energy effective theory in (2) *is* invariant under the corresponding transformation. The reason for this is that the normal ordering of the latter subtracts the symmetry-breaking terms. We may remove this accidental symmetry by explicitly introducing operators that break charge conjugation.